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EYE DIRECTION BY STEREO IMAGE PROCESSING USING CORNEAL REFLECTION ON AN IRIS

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ABSTRACT

The measurement systems of the movement of an eye direction are important in ophthalmology. In this paper, an eye direction algorithm for eye tracking is introduced by image processing in stereo cameras. We assume that iris images are observed by cameras. Our method uses the fact that the location of the corneal reflection on an iris region changes as the eye direction varies. The eye direction is estimated by image processing, based on the relation of the relative position of corneal reflection on the iris region.

KEY WORDS

Stereo image processing, corneal reflection, eye direction, genetic algorithm, image segmentation, contour grouping.

1 Introduction

Types of disability in vision are now detected by classifying the manners of nystagmus in ophthalmology in [1]. A relationship between smooth pursuit of eye movement and visual perception has been studied in [2], by observing disabled children. In the paper [2], they used an electrooculogram and a video at the same time. The electrooculogram measures electric potential differences between the nose and the ear, by applying voltage to them. The video records an eye movement. However, a detailed eye direction is not measured. So, it is important to develop a machine to determine an eye direction without injuring the eye by using CCD cameras.

In the authors' paper [3], moving triangles are recognized from the image ellipses of their inscribed circles in the triangles.

In the authors' paper [4], two eyes of a human inspector are analyzed for measurement of a movement of an eye direction, and we show the possibility of inspection by image processing in stereo cameras with computer programming. In order to have the same functions performed by human eyes, two cameras are used. The distance between two cameras is the same distance between the human eyes. Two cameras direct to the center of a pupil circle as the human eyes gaze the center. The eye direction is obtained by calculating the exterior product of the two vectors made from three points on the pupil circle. The three points on the pupil circle are calculated by applying author's theories to images taken by stereo cameras. Attitudes of the stereo cameras are adjusted to direct toward the center of pupil circle, then angle sensors give the attitude angles of the stereo cameras.

In this paper, we use the author's image processing method in [5]. As shown later, not only the iris but also the corneal reflection are extracted clearly in the image of faces by our method. However, the pupil is not extracted. Therefore, we need to replace the pupil by the iris on the theories in [4]. Furthermore, the radius of the iris is greater than that of the pupil, and so more accurate direction can be expected. We propose an algorithm for obtaining eye direction, which fits for image processing. The optimal attitudes of stereo cameras are determined from the locations of the corneal reflection images.

The paper [5] proposed a feature clustering method based on a genetic algorithm (GA) with an energy function for obtaining optimal segmentation. In the proposed algorithm, which we call MGA, the length of each genome is the number of features and each individual (genome) represents one assignment of the input-features to output layers. This algorithm first performs a sequence of operations for creating initial individuals(genomes) in the first generation. In this algorithm, we propose four different kinds of special mutation operations. This method is applied to determine our eye direction and is successful in recognizing the corneal reflection region with small curvature. This image processing is shown as an efficient method in measuring the eye direction.

In [6], an eye direction algorithm for the eye tracking was introduced. The iris images are observed by cameras. The method uses the fact that the location of the corneal reflection on the iris region changes as the eye direction varies. The eye direction is estimated by the relation of the relative position of corneal reflection on the iris region based on the affine projections. However, the affine projections do not give sufficient accuracy in the cases when the camera and the eye sphere are near each other.

In this paper, we use a perspective projection for calculating an eye direction. The relationship between the image of the corneal reflection and the image of the iris circle is solved for the perspective projections. In our algorithm, if the attitudes of two cameras are controlled to direct toward the center of the corneal radius, then the image of the corneal reflection moves to the origin in each camera coordinate and the rough eye direction is obtained. Therefore, to control the attitude of the cameras is necessary. Furthermore, we propose the procedure which improves its accuracy of eye direction from a rough eye direction without controlling the two cameras' attitudes. This procedure is called an 'epipolar chord procedure'.

2 Coordinate systems for object eye and stereo cameras

In this section, the eye sphere, the iris circle, and the locations and the angles of cameras are defined for coordinate systems of an object eye and stereo cameras.

2.1 World coordinate system

An object point in 'a world right hand coordinate system' is denoted as $\mathbf{X} = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$. Its origin lies at the center of the eye sphere, X axis is the horizontal axis, Y axis is the vertical axis and Z axis directs to the midpoint of the two camera locations.

2.2 Eye sphere

Assume that the radius r_0 of the eye sphere is $r_0 = 12mm$ and the radius of the iris circle is $a_0 = 5.5mm$ which are the average radii in the human eyes.

A half sphere point $\mathbf{q}(\beta, \gamma, r_0)$ in $Z \geq 0$ is expressed as

$$\mathbf{q}(\beta, \gamma, r_0) = r_0 \mathbf{u}_3 = r_0\ {}^t(\cos\gamma \sin\beta, \sin\gamma \sin\beta, \cos\beta),$$

where r_0 is the radius of sphere and "${}^t\ \cdot$ " denotes the transpose. $\mathbf{u}_3$ is the vector such that the point ${}^t(0, 0, r_0)$ on $Z - axis$ rotates around Y axis by the angle β in $0 \leq \beta < \frac{\pi}{2}$ and next around Z axis by the angle γ in $0 \leq \gamma < 2\pi$. That is, $\mathbf{u}_3$ the third column vector in the rotation matrix T, which is defined as

$$T = \begin{pmatrix} \cos\gamma\cos\beta & -\sin\gamma & \cos\gamma\sin\beta \\ \sin\gamma\cos\beta & \cos\gamma & \sin\gamma\sin\beta \\ -\sin\beta & 0 & \cos\beta \end{pmatrix} = (\mathbf{u}_1\mathbf{u}_2\mathbf{u}_3).$$

In this paper, eye direction is denoted by $\mathbf{u}_3$.

2.3 Iris circle

The iris circle with center $\mathbf{p} = \sqrt{r_0^2 - a_0^2}\mathbf{u}_3$ is defined as

$$a_0 \cos\psi \mathbf{u}_1 + a_0 \sin\psi \mathbf{u}_2 + \sqrt{r_0^2 - a_0^2}\mathbf{u}_3.$$

Note that the iris circle lies on the eye sphere and the center of the iris disc lies on the same direction with $\mathbf{q}$. The length from the center of the iris disc to the center of the eye sphere is $\sqrt{r_0^2 - a_0^2}$.

2.4 Corneal reflection

A "corneal reflection" is the image formed by the light reflected from the front convex surface of the cornea, which has about 7.8mm radius of curvature. The distance between the center of the corneal radius and the center of the eye sphere is 6mm.

2.5 Camera angles by the center of the corneal radius

Assume that each camera has its light, whose direction is the same with that of the camera. Each (right or left) camera is controlled so that the location of the corneal reflection image is its origin in the image space. Then both the right and left cameras direct to the common center of the corneal radius. In the following section, it is shown that the eye direction is obtained from the center location $\mathbf{h}$ of the corneal radius. The center location is given by $\mathbf{h} = h\mathbf{u}_3$, where $h = 6mm$.

2.6 Rotation matrices for cameras

The rotation matrices between two camera coordinates and the world coordinate, are defined such that the cameras are controlled to direct to the center of the corneal radius.

Let $\mathbf{O}_r = \begin{pmatrix} d \\ 0 \\ O_Z \end{pmatrix}$ be the location of the right camera and $\mathbf{O}_l = \begin{pmatrix} -d \\ 0 \\ O_Z \end{pmatrix}$ the location of the left camera. Here $2d$ is the distance of two cameras, which is the average distance between the human eyes and so $2d = 60mm$. O_Z is the distance from the center of the sphere to the mid point of the segment with two cameras and its value is fixed to $O_z = 300mm$. Let the right and left coordinates be attached to the lenses of the cameras as $\mathbf{X}_r = \begin{pmatrix} X_r \\ Y_r \\ Z_r \end{pmatrix}$ with the origin at the center of the right camera and $\mathbf{X}_l = \begin{pmatrix} X_l \\ Y_l \\ Z_l \end{pmatrix}$ with the origin at the center of the left camera. Here Z_r and Z_l direct to the center $\mathbf{h}$. Y_r and Y_l are axes perpendicular to the space constructed from the vectors $\mathbf{h} - \mathbf{O}_r$ and $\mathbf{h} - \mathbf{O}_l$ and so $Y_r = Y_l$. X_r is an axis perpendicular to (Z_r, Y_r) plane and X_l an axis perpendicular to (Z_l, Y_l) plane.

Then an object point $\mathbf{X}$ is

$$\mathbf{X} = R_r\mathbf{X}_r + \mathbf{O}_r; \mathbf{X} = R_l\mathbf{X}_l + \mathbf{O}_l.$$

Here R_r and R_l are 3×3 rotation matrices such that $R_r = [\mathbf{r}_1\mathbf{r}_2\mathbf{r}_3]$ and $R_l = [\mathbf{l}_1\mathbf{l}_2\mathbf{l}_3]$, where

$$\mathbf{r}_1 = \mathbf{r}_2 \times \mathbf{r}_3 = \frac{\mathbf{l}_3 \times \mathbf{r}_3}{\|\mathbf{l}_3 \times \mathbf{r}_3\|} \times \mathbf{r}_3, \mathbf{r}_2 = \mathbf{l}_2 = \frac{\mathbf{l}_3 \times \mathbf{r}_3}{\|\mathbf{l}_3 \times \mathbf{r}_3\|},$$

$$\mathbf{r}_3 = \frac{\mathbf{h} - \mathbf{O}_r}{\|\mathbf{h} - \mathbf{O}_r\|},$$

$$\mathbf{l}_1 = \frac{\mathbf{l}_3 \times \mathbf{r}_3}{\|\mathbf{l}_3 \times \mathbf{r}_3\|} \times \mathbf{l}_3, \mathbf{l}_3 = \frac{\mathbf{h} - \mathbf{O}_l}{\|\mathbf{h} - \mathbf{O}_l\|}.$$

3 Calculation of an object point $\mathbf{X}$ by using relation between right and left image points

'A perspective projection' is defined by the four equations

$$\mathbf{x}_r = \frac{1}{Z_r}\begin{pmatrix} X_r \\ Y_r \end{pmatrix}, \mathbf{x}_l = \frac{1}{Z_l}\begin{pmatrix} X_l \\ Y_l \end{pmatrix}.$$

Theorem [4]

1. The object point $\mathbf{X}$ is determined with respect to the image points $\mathbf{x}_r$ and $\mathbf{x}_l$ by $\mathbf{X} = B^{-1}\mathbf{d}$, where $B = \begin{pmatrix} \mathbf{x}_r^t\mathbf{r}_3 - {}^t\tilde{R}_r \\ {}^t\mathbf{l}_1 - {}^t\mathbf{l}_3 x_l \end{pmatrix}$ and $\mathbf{d} = \begin{pmatrix} -{}^t\tilde{R}_r\mathbf{O}_r + {}^t\mathbf{r}_3\mathbf{O}_r\mathbf{x}_r \\ ({}^t\mathbf{l}_1 - {}^t\mathbf{l}_3 x_l)\mathbf{O}_l \end{pmatrix}$. Here $\mathbf{x}_r^t\mathbf{r}_3 = \begin{pmatrix} x_r r_{31} & x_r r_{32} & x_r r_{33} \\ y_r r_{31} & y_r r_{32} & y_r r_{33} \end{pmatrix}$ with ${}^t\mathbf{r}_3 = (r_{31}, r_{32}, r_{33})$ and ${}^t\tilde{R}_r = \begin{pmatrix} {}^t\mathbf{r}_1 \\ {}^t\mathbf{r}_2 \end{pmatrix}$.

2. The "Longuet-Higgin's relation" [7] between the left image point $\begin{pmatrix} x_l \\ y_l \end{pmatrix}$ and the given right image point $\mathbf{x}_r$ is obtained as follows:

$$y_l = \frac{a_0(\mathbf{x}_r)x_l + a_1(\mathbf{x}_r)}{a_2(\mathbf{x}_r)x_l + a_3(\mathbf{x}_r)}.$$

   Coefficients $a_i(\mathbf{x}_r)(i = 0, 1, 2, 3)$ depend on only camera angles and known parameters: $a_0(\mathbf{x}_r) = {}^t\mathbf{l}_2(\mathbf{b}_2 - \mid B_2 \mid \mathbf{O}_l)$, $a_1(\mathbf{x}_r) = {}^t\mathbf{l}_2(\mathbf{b}_1 - \mid B_1 \mid \mathbf{O}_l)$,

$$a_2(\mathbf{x}_r) = -{}^t\mathbf{l}_3\mathbf{O}_l \mid B_2 \mid + {}^t\mathbf{l}_3\mathbf{b}_2,$$

$$a_3(\mathbf{x}_r) = -{}^t\mathbf{l}_3\mathbf{O}_l \mid B_1 \mid + {}^t\mathbf{l}_3\mathbf{b}_1.$$

   Here

$$\mathbf{b}_1 = B_1^c(\mathbf{d}_1 + \mathbf{d}_2), \mathbf{b}_2 = B_1^c\mathbf{d}_3 + B_2^c\mathbf{d}_1,$$

$$B_1 = \begin{pmatrix} \mathbf{x}_r{}^t\mathbf{r}_3 - {}^t\tilde{R}_r \\ {}^t\mathbf{l}_1 \end{pmatrix}, B_2 = \begin{pmatrix} \mathbf{x}_r{}^t\mathbf{r}_3 - {}^t\tilde{R}_r \\ -{}^t\mathbf{l}_3 \end{pmatrix},$$

   $\mathbf{d}_1 = \begin{pmatrix} -{}^t\tilde{R}_r\mathbf{O}_r + {}^t\mathbf{r}_3\mathbf{O}_r\mathbf{x}_r \\ 0 \end{pmatrix}$, $\mathbf{d}_2 = \begin{pmatrix} \mathbf{0} \\ {}^t\mathbf{l}_1\mathbf{O}_l \end{pmatrix}$, $\mathbf{d}_3 = -\begin{pmatrix} \mathbf{0} \\ {}^t\mathbf{l}_3\mathbf{O}_l \end{pmatrix}$,
   where $\mid M \mid$ denotes the determinant and suffix M^c denotes the cofactor matrix.

4 Initial procedure

Since the images of the center location $\mathbf{h}$ of the corneal radius are $\mathbf{x}_r = \mathbf{0}$ and $\mathbf{x}_l = \mathbf{0}$, the eye direction $\mathbf{u}_3$ is given by the formula: $\mathbf{u}_3 = \frac{B_0^{-1}\mathbf{d_0}}{\|B_0^{-1}\mathbf{d_0}\|}$, where $B_0 = \begin{pmatrix} -{}^t\tilde{R}_r \\ {}^t\mathbf{l}_1 \end{pmatrix}$, $\mathbf{d_0} = \begin{pmatrix} -{}^t\tilde{R}_r\mathbf{O}_r \\ {}^t\mathbf{l}_1\mathbf{O}_l \end{pmatrix}$. This is proved from that

$${}^t\tilde{R}_r(\mathbf{h} - \mathbf{O}_r) = \mathbf{0}, {}^t\mathbf{l}_1(\mathbf{h} - \mathbf{O}_l) = 0,$$

if $\mathbf{x}_r = \mathbf{0}$ and $\mathbf{x}_l = \mathbf{0}$, and that $\mathbf{h} = h\mathbf{u}_3$ is the unique solution of the above equations. In the initial procedure, we calculate the eye direction $\mathbf{u}_3$ by $\frac{B_0^{-1}\mathbf{d_0}}{\|B_0^{-1}\mathbf{d_0}\|}$.

If the camera directs to the center of the corneal radius exactly, the correct eye direction is calculated by the initial procedure.

However, the camera only directs in the neighborhood of the center $\mathbf{h}$ of the corneal radius, since the cameras are controlled directing to the corneal reflection by trial and error with human hand, whether the image data of the corneal reflection lies on the origins in two camera coordinates or not. So the center $\mathbf{h}$ would be calculated with error by the initial procedure. In the next procedure, the attitudes of the cameras are adjusted directing to the correct $\mathbf{h}$ by calculating correct $\mathbf{u}_3$ and then putting $\mathbf{h} = h\mathbf{u}_3$. This epipolar chord procedure is shown in the following sections 5, 6 and 7.

5 Image of the iris circle

Let $\mathbf{v}$ be $a_0\begin{pmatrix} \cos\psi \\ \sin\psi \end{pmatrix}$. Substituting $\mathbf{X} = (\mathbf{u}_1, \mathbf{u}_2)\mathbf{v} + \mathbf{p}$ into $\mathbf{X}_r = {}^tR_r(\mathbf{X} - \mathbf{O}_r)$, we have $\mathbf{X}_r = \begin{pmatrix} A_r\mathbf{v} + \mathbf{w}_r \\ {}^t\mathbf{a}_r\mathbf{v} + \zeta_r \end{pmatrix}$. A transformation from the iris circle to the right image is expressed as $\mathbf{x}_r = \frac{A_r\mathbf{v} + \mathbf{w}_r}{{}^t\mathbf{a}_r\mathbf{v} + \zeta_r}$, where $A_r = \begin{pmatrix} {}^t\mathbf{r}_1(\mathbf{u}_1\mathbf{u}_2) \\ {}^t\mathbf{r}_2(\mathbf{u}_1\mathbf{u}_2) \end{pmatrix}$, $\mathbf{w}_r = \begin{pmatrix} {}^t\mathbf{r}_1(\mathbf{p} - \mathbf{O}_r) \\ {}^t\mathbf{r}_2(\mathbf{p} - \mathbf{O}_r) \end{pmatrix}$ ${}^t\mathbf{a}_r = {}^t\mathbf{r}_3(\mathbf{u}_1\mathbf{u}_2)$, and $\zeta_r = {}^t\mathbf{r}_3(\mathbf{p} - \mathbf{O}_r)$. A transformation to the left image is expressed as $\mathbf{x}_l = \frac{A_l\mathbf{v} + \mathbf{w}_l}{{}^t\mathbf{a}_l\mathbf{v} + \zeta_l}$, where $A_l = \begin{pmatrix} {}^t\mathbf{l}_1(\mathbf{u}_1\mathbf{u}_2) \\ {}^t\mathbf{l}_2(\mathbf{u}_1\mathbf{u}_2) \end{pmatrix}$, $\mathbf{w}_l = \begin{pmatrix} {}^t\mathbf{l}_1(\mathbf{p} - \mathbf{O}_l) \\ {}^t\mathbf{l}_2(\mathbf{p} - \mathbf{O}_l) \end{pmatrix}$ ${}^t\mathbf{a}_l = {}^t\mathbf{l}_3(\mathbf{u}_1\mathbf{u}_2)$ and $\zeta_l = {}^t\mathbf{l}_3(\mathbf{p} - \mathbf{O}_l)$.

6 Epipolar plane and epipolar chord

6.1 Center h of corneal radius, corneal reflection h_C and iris circle

Epipolar geometry is defined as follows in [8].

Definition [8] The plane generated from the three points, i.e., the two camera locations and the object point $\mathbf{X}$, is called an 'epipolar plane'. An intersection line of the epipolar plane and each image plane of cameras is called an 'epipolar line'. The epipolar lines for several objects intersect at one point which is called an 'epipole' and constitute a fan-shaped figure. This particular geometry such as the fan-shaped figure is called an 'epipolar geometry'. The epipole in the image by the right (left) camera is the image of the left (right) camera location. It is denoted as

$\mathbf{e}_r$ (or $\mathbf{e}_l$) [8]. Set the object point $\mathbf{X}$ be the center $\mathbf{h}$ of the corneal radius in this definition. Then a special epipolar chord is defined as follows, which lies on the special epipolar line passing through the image of $\mathbf{h}$. Consider an epipolar plane, which is constructed from the three points, i.e., two camera locations $\mathbf{O}_r$,$\mathbf{O}_l$ and the center $\mathbf{h}$ of the corneal radius. A point in the epipolar plane is expressed as $\mathbf{Q} = \alpha_1(\mathbf{O}_l - \mathbf{h}) + \alpha_2(\mathbf{O}_r - \mathbf{h}) + \mathbf{h}$, where α_1 and α_2 take real scalar values.

Definition 'An epipolar chord' is defined as a segment $[\mathbf{f}_{r-}, \mathbf{f}_{r+}]$ (or $[\mathbf{f}_{l-}, \mathbf{f}_{l+}]$) projected from the specified chord $[\mathbf{f}_-, \mathbf{f}_+]$ on the intersection of the epipolar plane and iris disc.

Then the epipolar chord lies on the epipolar line which connects the image of the corneal reflection $\mathbf{h}_r$ (or $\mathbf{h}_l$) on the iris image and the epipoles $\mathbf{e}_r$ (or $\mathbf{e}_l$).

If the object iris circle lies on the epipolar plane, then the image of the iris circle concentrates into the epipole line and the 'epipolar chord procedure' in this paper does not work.

The epipolars $\mathbf{e}_r$, $\mathbf{e}_l$, the intersection line on the epipolar plane and the plane of the iris disc are given in the following lemma. Proof is obtained by roots of two degree polynomials and is omitted here.

Lemma

1. The epipoles are determined only with respect to the camera angles and are given as $\mathbf{e}_r = \binom{r_1^1/r_3^1}{0}/, \mathbf{e}_l = \binom{l_1^1/l_3^1}{0}/$, where r_i^1 is the first element in $\mathbf{r}_i$ and l_i^1 is the first element in $\mathbf{l}_i$ for $i = 1, 2, 3$.

2. Let $\mathbf{Q}$ be a point in an epipolar plane : $\mathbf{Q} = \alpha_1(\mathbf{O}_l - \mathbf{h}) + \alpha_2(\mathbf{O}_r - \mathbf{h}) + \mathbf{h}$, where $\mathbf{h}$ is the center of the corneal radius. The iris circle on the eye sphere at $\mathbf{p}$ is defined as $v_1\mathbf{u}_1 + v_2\mathbf{u}_2 + \mathbf{p}$, where $\mathbf{p} = \sqrt{r_0^2 - a_0^2}\mathbf{u}_3$. Here $\mathbf{u}_1$ and $\mathbf{u}_2$ are tangent vectors at $r_0\mathbf{u}_3$ on the sphere and $v_1 = a_0 \cos\psi, v_2 = a_0 \sin\psi$.
   Then the specified chord $[\mathbf{f}_-, \mathbf{f}_+]$ on the intersection of the epipolar plane and the iris disc are given as follows.
   Put $o_{ri} =^t \mathbf{O}_r\mathbf{u}_i$, $o_{li} =^t \mathbf{O}_l\mathbf{u}_i$ for $i = 1, 2, 3$,
   $$a_1 = \frac{h - o_{r3}}{o_{l3} - h} o_{l1} + o_{r1}, b_1 = \frac{(\sqrt{r_0^2 - a_0^2} - h)o_{l1}}{o_{l3} - h},$$
   $$a_2 = \frac{h - o_{r3}}{o_{l3} - h} o_{l2} + o_{r2}, b_2 = \frac{(\sqrt{r_0^2 - a_0^2} - h)o_{l1}}{o_{l3} - h}.$$
   The edges in specified chord on the iris circle are given as
   $$\mathbf{f}_\pm = a_0(\cos\psi_\pm \mathbf{u}_1 + \sin\psi_\pm \mathbf{u}_2) + \mathbf{p},$$
   where the angle $\psi_\pm$ of the specified chord is given as
   $$\psi_\pm = \arcsin(\frac{c_1 \pm \sqrt{c_2}a_2}{a_0(a_1^2 + a_2^2)}),$$
   $c_1 = -a_1b_1a_2 + a_1^2b_2$ and $c_2 = a_0^2(a_1^2 + a_2^2) - (a_1b_2 - a_2b_1)^2$ for β and γ.

6.2 Corneal reflection and edge points of epipolar chords

In the following theorem, we give relations among the edges points of epipolar chords, the images of corneal reflections and the image of the center $\mathbf{h}$ of corneal radius.

Theorem Assume :

1. the right and left cameras direct to the center of corneal radius $\mathbf{h}$,

2. two rays of light come from the right and left cameras respectively. The ray direction is the same as the camera direction.

Then,

1. both images of the corneal reflection $\mathbf{h}_{cl}$ (res. $\mathbf{h}_{cr}$) and the center $\mathbf{h}_l$ (res. $\mathbf{h}_r$) of corneal radius by the left (res. right) camera lie at the common origin of its image plane, i.e., at $\mathbf{x}_l = \binom{0}{0}$, (res. $\mathbf{x}_r = \binom{0}{0}$),

2. edge points of the epipolar chords $[\mathbf{f}_{r-}, \mathbf{f}_{r+}]$ and $[\mathbf{f}_{l-}, \mathbf{f}_{l+}]$ are given by $\psi_\pm$ as
   $$\mathbf{f}_{r\pm} = \frac{A_r\mathbf{v}_\pm + \mathbf{w}_r}{{}^t\mathbf{a}_r\mathbf{v}_\pm + \zeta_r}, \mathbf{f}_{l\pm} = \frac{A_l\mathbf{v}_\pm + \mathbf{w}_l}{{}^t\mathbf{a}_l\mathbf{v}_\pm + \zeta_l},$$
   where $\mathbf{v}_\pm = a_0\binom{\cos\psi_\pm}{\sin\psi_\pm}$,

3. the epipolar chord $[\mathbf{f}_{r-}, \mathbf{f}_{r+}]$ (res. $[\mathbf{f}_{l-}, \mathbf{f}_{l+}]$) lie on the line $y_r = 0$ (res. $y_l = 0$), and edge points $\mathbf{f}_{r-}$ and $\mathbf{f}_{r+}$ (res. $\mathbf{f}_{l-}$ and $\mathbf{f}_{l+}$) lie on the line $y_r = 0$ (res. $y_l = 0$).

7 Epipolar chord procedure

In this section, we introduce the epipolar chord procedure by using pixel images of the cameras. Assume that we obtain the distance $2d$ of two cameras, the distance O_z of the mid point between two cameras, the center of the eye sphere, the radius r_0 of the eye sphere, the radius a_0 of the iris circle and the distance h of the corneal radius center $\mathbf{h}$ from $\mathbf{O}$. Suppose contour images of the iris circle are given on the two pixel images of two cameras. Let $\mathbf{y}_r$ be the variable of the right pixel image points and $\mathbf{y}_l$ the variable of the left pixel image points.

The eye direction $\mathbf{u}_3$ is calculated by the following epipolar chord procedure for the pixel image points after calculating (β, γ) by the initial procedure.

1. *Select the optimal points* $\mathbf{f}_{ri}$ *and* $\mathbf{f}_{li}$.
   The angle ψ_0 is given by $\psi_0 = \psi_+ = \arcsin(\frac{c_1+\sqrt{c_2}a_2}{a_0(a_1^2+a_2^2)})$, where $c_1 = -a_1b_1a_2 + a_1^2b_2$ and $c_2 = a_0^2(a_1^2 + a_2^2) - (a_1b_2 - a_2b_1)^2$ for β and γ. Calculate the right and the left image points $\mathbf{f}_r^{(i)} = \frac{A_r\mathbf{v}_i+\mathbf{w}_r}{{}^t\mathbf{a}_r\mathbf{v}_i+\zeta_r}$ and $\mathbf{f}_l^{(i)} = \frac{A_l\mathbf{v}_i+\mathbf{w}_l}{{}^t\mathbf{a}_l\mathbf{v}_i+\zeta_l}$, by using the points $\mathbf{v}_i = a_0\binom{\cos(\psi_0+\frac{2\pi i}{3})}{\sin(\psi_0+\frac{2\pi i}{3})}$ for $i = 0, 1, 2$.

2. *Select the practical image points* $\mathbf{y}_{ri}$ *nearest to the image of the optimal points* $\mathbf{f}_r^{(i)}$.

   Let $\mathbf{y}_{ri}$ be the point to minimize $\mid \mathbf{y}_r - \mathbf{f}_r^{(i)} \mid$ in the contour of the image ellipse for $i = 0, 1, 2$. Here $\mid \mathbf{x} \mid$ denotes the sum of absolute values of the elements in $\mathbf{x} = \binom{x}{y}$: $\mid \mathbf{x} \mid = \mid x \mid + \mid y \mid$.

3. *Select the practical left image points* $\mathbf{y}_{li}$.

   If the camera directs not to the center $\mathbf{h}$, then the radius $\mathbf{v}_0$ has the error depending on (β, γ). The left image points are assigned by the Longuet Higgin's relation. This relation for each left image point may give two solutions. In order to select one of these, we choose one point which is near to each left image point $\mathbf{f}_l^i$ and calculate the point $\mathbf{y}_{(li)}$ for minimizing $\mid y_l - \frac{a_0(\mathbf{y}_{ri})x_l + a_1(\mathbf{y}_{ri})}{a_2(\mathbf{y}_{ri})x_l + a_3(\mathbf{y}_{ri})} \mid + \|\mathbf{y}_l - \mathbf{f}_l^{(i)}\|$ $(i = 0, 1, 2)$.

4. *Three points* $\mathbf{X}_i$ *on iris circle.*

   The three points $\mathbf{X}_i$ on the object iris circle are calculated from the three pairs of image points $(\mathbf{y}_{ri}, \mathbf{y}_{li})$ by substituting $(\mathbf{x}_r, \mathbf{x}_l) = (\mathbf{y}_{ri}, \mathbf{y}_{li})$ into $\mathbf{X}_i = B^{-1}\mathbf{d}$, where $B = \binom{\mathbf{x}_r {}^t\mathbf{r}_3 - {}^t\tilde{R}_r}{{}^t\mathbf{l}_1 - {}^t\mathbf{l}_3 x_l}$, $\mathbf{d} = \binom{-{}^t\tilde{R}_r\mathbf{O}_r + {}^t\mathbf{r}_3\mathbf{O}_r\mathbf{x}_r}{({}^t\mathbf{l}_1 - {}^t\mathbf{l}_3 x_l)\mathbf{O}_l}$.

5. *Eye direction* $\mathbf{u}_3$.

   Calculate the exterior product $\mathbf{X} = (\mathbf{X}_2 - \mathbf{X}_1) \times (\mathbf{X}_0 - \mathbf{X}_1)$. If $\mid \frac{3\sqrt{3}}{2}a_0^2 - \|\mathbf{X}\| \mid < \epsilon$ then stop the procedure and if $\mid \frac{3\sqrt{3}}{2}a_0^2 - \|\mathbf{X}\| \mid > \epsilon$ then calculate $\mathbf{u}_3 = \frac{\mathbf{X}_2 - \mathbf{X}_1}{\|\mathbf{X}_2 - \mathbf{X}_1\|} \times \frac{\mathbf{X}_0 - \mathbf{X}_1}{\|\mathbf{X}_0 - \mathbf{X}_1\|}$. Adjust the camera angles so as the corneal reflection moves to the origin of the respective camera image coordinate.

   Repeat the initial procedure in the section 4 and epipolar chord procedure in this section, by putting $\mathbf{p} = \sqrt{r_0^2 - a_0^2}\mathbf{X}$ and $\mathbf{h} = h\mathbf{X}$.

   Here ϵ is taken as a small value depending on the accuracy of the precision of the computer programming, the pixel accuracy in the camera image and the accuracy of the camera angles measured by the camera angle sensors.

8 Image Segmentation based on Genetic algorithm

For practical usage, we need real eye image segmentation. In computer vision, image segmentation is fundamental, but is still difficult. The progress in its solution has not kept up with the increased demand. Therefore, a number of studies have been conducted in the past. For our purpose, we need to separate the contour of an iris, a corneal reflection from real eye images. In this paper, we use modified genetic algorithm (MGA) with an energy function [5], [9]. This algorithm is effective for contour grouping applications. In the algorithm, the energy function consists of local competitive and cooperative interactions among features and a global inhibition, which is similar to the one used in the competitive-layer model (CLM) [10]. The difference from the CLM is that our energy function does not include an annealing term. By using a genetic algorithm (GA), it removes the difficulties of annealing problem and parameter adjustments which are necessary in the CLM. Our energy function is used as the fitness function of GA. We use the GA for this problem, because the GA has been applied successfully to many searching problems for an approximate global minimum of objective functions. The MGA first performs a sequence of operations to create initial individuals (first generation) using the data of local interactions. These operations obtain the configuration of features that corresponds to the local energy minimum. After this initial processing, we apply the GA operations together with four special mutations in order to obtain the configuration of features that globally minimizes the energy function.

For contour grouping, each feature is represented by its pixel's position and its local edge orientation. Edges with similar orientation in the small neighborhood share positive local interactions with each other. On the other hand, almost right-angled edges with each other in the small neighborhood have negative local interactions. By our program, features having positive local interactions with each other tend to group in the same layer.

- Figure 1 is a real image of a pupil taken by a digital camera.
- Figure 2 (a) is the image after noise removal and Figure 2 (b) is the image of binarization.
- We use Prewitt method to detect edges: Figure 3 (a).
- We choose only one edge feature having maximum intensity grayness in each 3×3 pixel square. Then, the number of edge features M is reduced to $M/9$: Figure 3 (b).

As a result of MGA, the original image is separated into 7 layers: the corneal reflection in Fig. 4 (a), the contour of the iris in Fig. 4 (b), the upper line in a double edged eyelid and the lower curve line in a upper eyelid in Fig. 4 (c) and the lower curve line in the eyelashes in Fig. 4 (d). The others in Fig.4 (e),(f),(g) are noise layers .

9 Conclusion

It is shown that the MGA method extracts the images of corneal reflection and iris contour from the image of the human face. Therefore, the initial rough eye direction is given by the corneal reflection, whose location is given by the image processing of the human face. The images of corneal reflection and iris contour are used for calculating the accurate eye direction by the epipolar chord method. Thus the MGA method plays important role in calculating the eye direction.

Figure 1. Eye image taken by a digital camera:200 $\times$ 200 pixels.

(a) (b)

Figure 2. (a) Eye image after noise removal (b) Binarization

(a) (b)

Figure 3. (a) Detect edges of eye image. (b) Eye image after pixel reduction

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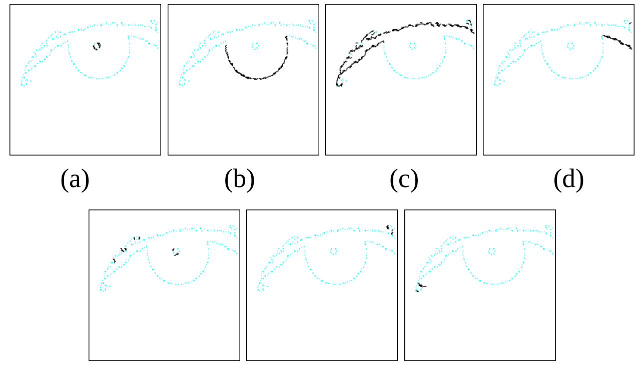

Figure 4. Separation of the eye image

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Appendix Numerical results

Numerical tests are performed in the case $(\beta, \gamma) = (30°, 0.00001°)$. The 100 pixel image points on the iris image ellipse contour are given by the image points of the equal distance points on the iris circle. ψ_0 is computed as nearly zero i.e. $-2.255e-007$ as desired. The 100 pixel image points are numbered starting from number 0 of the point with angle ψ_0. The numers 0, 33 and 67 are selected for both the points $\mathbf{y}_{ri}$ and $\mathbf{y}_{li}$. From the image pairs $(\mathbf{y}_{ri}, \mathbf{y}_{li})$, the three $\mathbf{X}_i$ $(i = 0, 1, 2)$ constitute the vertices of a regular triangle. The area of this regular triangle has the error $\epsilon = 4.029183e-002$. Accurate eye direction $\mathbf{u}_3$ is obtained by the initial and epipolar chord procedures.

Table Numerical results

Initial procedure to compute center $\mathbf{h}$ of the corneal radius			
${}^t\mathbf{r}_1$	-9.958e-001	-1.619e-010	9.120e-002
${}^t\mathbf{r}_2$	0.000e+000	1.000e+000	1.776e-009
${}^t\mathbf{r}_3$	-9.120e-002	1.768e-009	-9.958e-001
${}^t\mathbf{l}_1$	-9.937e-001	1.975e-010	-1.112e-001
${}^t\mathbf{l}_2$	0.000e+000	1.000e+000	1.776e-009
${}^t\mathbf{l}_3$	1.112e-001	1.765e-009	-9.937e-001
Computed $\mathbf{h}$	3.000e+000	5.235e-007	5.196e+000
Computed $\mathbf{u}_3$	5.000e-001	8.726e-008	8.660e-001
Epipolar Chord procedure to compute eye direcion $\mathbf{u}_3$			
$\mathbf{X}_0$	1.049e+001	4.871e-007	6.257e+000
$\mathbf{X}_1$	2.846e+000	5.221e+000	1.067e+001
$\mathbf{X}_2$	2.846e+000	-5.221e+000	1.067e+001
Computed $\mathbf{u}_3$	5.000e-001	8.726 e-008	8.660e-001